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AGRICULTURAL MEMORANDUM NO 532

Number of days of air frost in the winter months (December, January, February) in Scotland in relation to the mean air temperature for the same period - on a period average basis, and for individual years at selected stations

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Summary

The relationships between mean daily temperature ($\bar{T}$) averaged over the winter months (December, January and February) and the corresponding number of air frosts (N) were examined

(i) on a period average basis, mainly 1960/61 to 1969/70 for some 19 stations in Scotland;

and (ii) on a winter by winter basis for 2 stations Edinburgh (Blackford Hill Observatory) and Eskdalemuir Observatory (Dumfriesshire) for up to 30 years.

For case (i) the least squares best fit - valid over the range $32^{o}F < \bar{T} < 42^{o}F$ - was

$$N = 230 - 5.10\bar{T} \qquad (r^2 = 0.962)$$

and the best second-degree curve ($20^{o}F < \bar{T} < 42^{o}F$)

$$N = 32.1 + 5.97\bar{T} - 0.154\bar{T}^2 \qquad (R^2 = 0.972)$$

The error for individual stations was less than 3 days for all but 3 of the stations (maximum error 6). This second equation was found to fit a selection of stations in Iceland very closely.

For case (ii), linear regressions were computed; that for Edinburgh (1940/41 to 1970/71) being

$$N = 241 - 5.38\bar{T} \qquad (r^2 = 0.808)$$

Use of this equation for other stations in South Scotland showed that in individual years errors of as much as ±10 days occur from time to time, but most discrepancies lie within the range ±5 days.

Introduction

Place to place comparisons, interpolation and extrapolation in time and space, can be rendered more reliable if statistical relationships between allied series of data can be appealed to.

A long-term average of mean daily temperature (on a monthly basis) can now be estimated for any site in Scotland with some confidence; the corresponding estimate of days with frost deduced directly from frost frequencies reported from other stations entails a greater degree of uncertainty. This contribution deals with the numerical relationship between frost frequency in winter and the corresponding mean daily temperature.

Attention was concentrated upon the winter period viz December, January and February, the data assembled being:

(1) period averages (as far as possible for the 10 winters 1960/61 to 1969/70) of mean daily temperature and mean number of frosts at some 20 stations in Scotland;

(2) similar data, but for individual winters for Eskdalemuir Observatory and Edinburgh (Blackford Hill Observatory).

Clearly any result will be affected by the magnitude of the mean diurnal range and hence can only be valid for an area in which the mean diurnal fluctuation is sensibly uniform.

Analysis

(a) Period average results

In Figure 1 period average data (mainly for the winters of 1960/61 to 1969/70) of mean daily temperature and mean number of air frosts for some 19 stations (these are listed below) are plotted together with the best-fitting first and second degree curves.

Number		Ht above MSL (ft)	Period
1	Coire Cas (Cairngorms nr Aviemore)	2500	5 yrs within 1963/64–1969/70
2	Braemar	1113	1960/61–1969/70
3	Glenmore Lodge	1120	9 yrs within 1960/61–1969/70
4	Grantown-on-Spey	725	7 yrs within 1960/61–1969/70
5	Dalwhinnie	1176	1956/57–1960/61
6	Carnwath (nr Carstairs, Lanarks)	706	1960/61–1969/70
7	West Linton Peeblesshire)	800	1960/61–1969/70
8	Eskdalemuir Observatory (Dumfriesshire)	794	1960/61–1969/70
9	Faskally (Pitlochry)	310	1960/61–1969/70
10	Perth	77	1960/61–1969/70
11	Dyce (Aberdeen)	190	1960/61–1969/70
12	Mylnefield (nr Dundee)	98	1960/61–1969/70
13	Nairn	20	1961/62–1969/70
14	Edinburgh (Royal Botanic Garden)	85	1956/57–1969/70
15	Renfrew (Glasgow)	26	1960/61–1969/70
16	Wick	118	1960/61–1969/70
17	Auchincruive (nr Ayr)	147	1960/61–1969/70
18	Benbecula (Outer Hebrides)	19	1960/61–1969/70
19	Tiree	29	1960/61–1969/70

Obviously the maximum possible number of days with frost in any winter is 90 (31 + 31 + 28) and the straight line cannot be extrapolated to temperatures lower than about 30 or 32°F (about 0°C): furthermore the curve must reach

its apex of 90 days at some mean temperature well below 0°C and thereafter remain at 90 days.

If

N = average number of days of frost per winter

$\bar{T}$ = average mean daily air temperature (°F)

then

$N = 230 - 5.10\bar{T}$ (best straight line)

$N = 32.1 + 5.97\bar{T} - 0.154\bar{T}^2$ (best second degree curve)

with respectively a linear correlation coefficient of −0.981 and a multiple correlation coefficient of 0.985.

The apex of the parabola reaches 89.8 ($\simeq 90$) at a temperature of 19.4°F. This suggests that if mean daily temperature during the winter is below about 20°F every winter night will be one with air frost.

As an independent test for the equations, data for some 9 stations in Iceland (i.e. a region subject to a maritime climate) were extracted. The results listed below show that parabola predicts the frost frequency to within an error of at most one unit.

Station	Lat N	Long W	Mean "winter" temperature	Mean number of days with frost	
				Predicted	Actual
Blönduós	65°40'	20°18'	29.3	74.7	74
Húsavik	66°02'	17°21'	30.4	71.2	71
Hladhamar	65°16'	21°10'	28.4	77.3	77
Nautabu	65°27'	19°22'	27.7	79.1	79
Reykjahlid	65°39'	16°55'	25.3	84.5	84
Grimsstadir	65°38'	16°07'	24.1	86.4	86
Möðrudalur	65°22'	15°53'	21.4	89.3	90
Gunnhildargerdi	65°33'	14°23'	28.7	76.3	76
Pingvellir	64°15'	21°07'	28.6	76.7	76

(b) Individual winters at Eskdalemuir and Edinburgh and three other stations in southern Scotland

The data and best straight line for Eskdalemuir and Edinburgh are set out in Figs 2 and 3. For the latter station the complete series 1940/41 to 1970/71 has been split into two sub-series viz 1940/41 to 1954/55 and 1955/56 to 1970/71; Eskdalemuir data are for the later period only.

Clearly from the scatter diagram all the observations lie within the range for which a linear relationship can be accepted. Expressing the relationship in the form:

N (= No. of frosts) = $a + b\bar{T}$ (= mean winter temperatures)

then the statistical parameters for the several straight lines are as under:

Place	Period	a	b	r (corr. coeff.)
Edinburgh	1940/41	234	-5.16	-0.946
(Royal Observatory)	to 1954/55			
	1955/56	260	-5.91	-0.869
	to 1970/71			
	1940/41	241	-5.38	-0.899
	to 1970/71			
Eskdalemuir	1955/56	204	-4.38	-0.891
	to 1970/71			

On the long-period average basis discussed in (a) above, the respective coefficients are 230 and -5.10.

When all the straight lines are plotted together, they are found to occupy a narrow band. To the extent that the lines are judged identical it can be stated that, in respect of winter frosts, the area is homogeneous as regards variations in space and in time.

As expected the errors of estimate of data relating to individual winters exhibit a greater degree of scatter than for the long period values.

The standard errors of estimate are:

(i) long-period average data (see Fig 1) 2.7 days

(ii) individual winters

Edinburgh (Blackford Hill) See Fig 2

1940/41 to 1954/55 3.7 "

1955/56 to 1970/71 5.5 "

1940/41 to 1970/71 4.9 "

Eskdalemuir (Fig 3) 1955/56 to 1970/71 4.4 "

For a selection of individual winters, the errors (i.e. estimated-actual) are given below. Rather surprisingly the largest errors in the 10 winters 1960/61 to 1969/70 are for Blackford Hill Observatory in spite of the fact that the linear regression was based upon data for that station albeit for the 31 winters 1940/41 to 1970/71.

Table 1/ indicates the magnitude of the errors.

Table 1. Differences from actual frost frequencies during the winter, of estimates based upon mean temperature for the same period; derived from linear regression based upon data for Edinburgh (Blackford Hill Observatory. 1940/41 to 1970/71): for selected stations and period 1960/61 to 1969/70

	1960 -61	1961 -62	1962 -63	1963 -64	1964 -65	1965 -66	1966 -67	1967 -68	1968 -69	1969 -70
Edinburgh Blackford Hill	0	0	−4	+7	+9	+6	+7	−5	+6	+5
Edinburgh Botanic Garden	−5	−4	+0	+1	+3	−1	+2	−9	0	+2
Mylnefield (Dundee)	−3	+1	0	+5	−4	+2	+8	−6	−1	+1
Abbotsinch (Glasgow)	−5	0	−2	−3	−6	+1	0	−8	+4	+1

For individual winters therefore, an error of estimate of up to ±10 days in frost frequency can occur although most discrepancies will lie within the range ± 5 days. For the period average analysis based upon the linear regression, an error as great as ±5 days is exceptional.

A more detailed scrutiny of the data at least suggests that stations could be rationally grouped into:

(i) those at markedly maritime site or freely exposed on hill sides

(ii) those which are surrounded by rather higher land, even at a considerable distance, and whose site may be judged to tend to have some of the characteristics of a frost hollow.

In the latter case there are, on a period average basis, more frosts during the winter for a given average mean temperature, than for case (i) - this is physically acceptable: a difference of between 5 and 10 days could be provisionally suggested.

RE 19589 TEL(9)

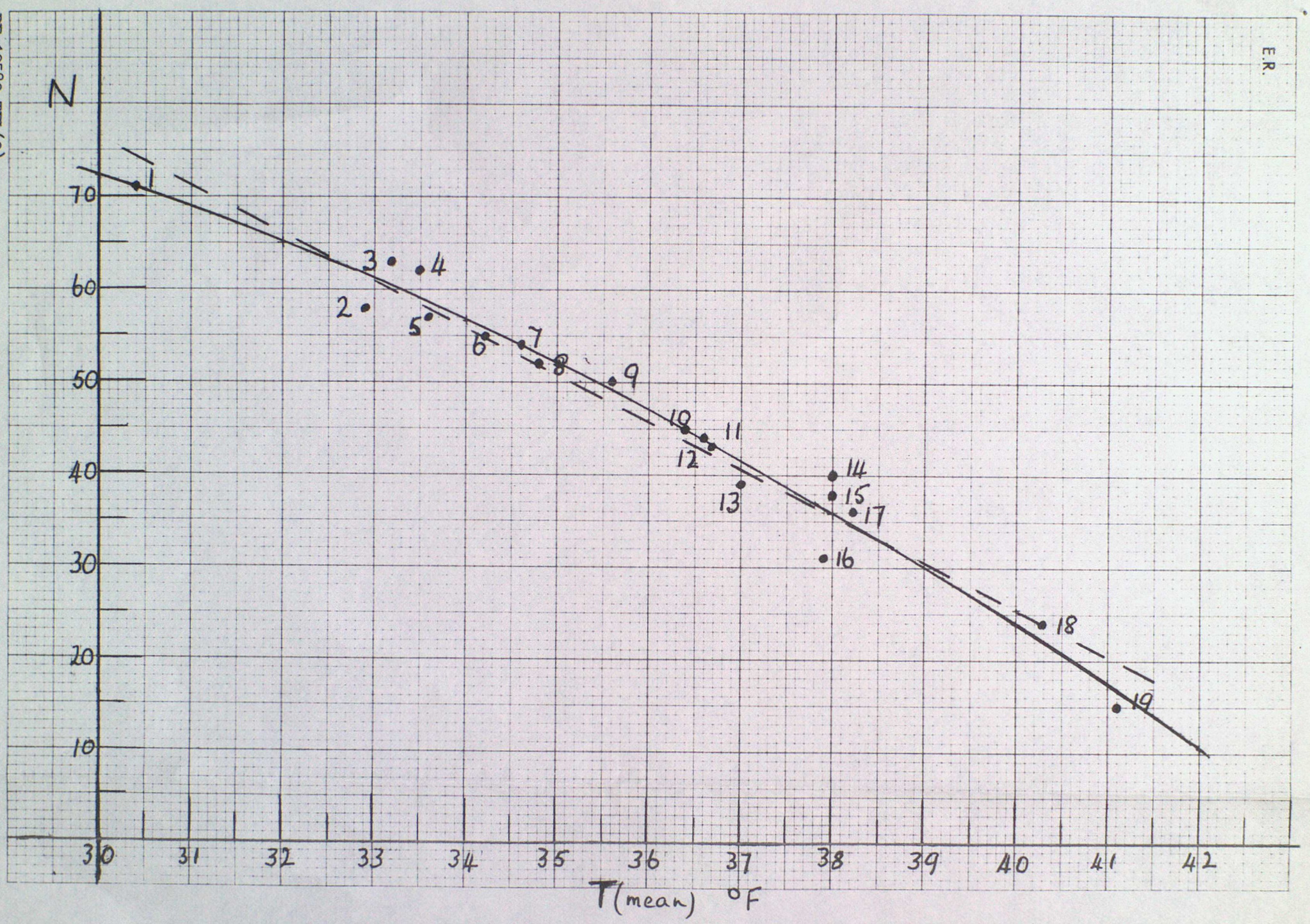

Fig 1 Relationship between average number of air frosts (N) and average mean daily air temperature for the winter months (December-February incl).
For identification of stations, see text.

RE 19589 TEL(11)

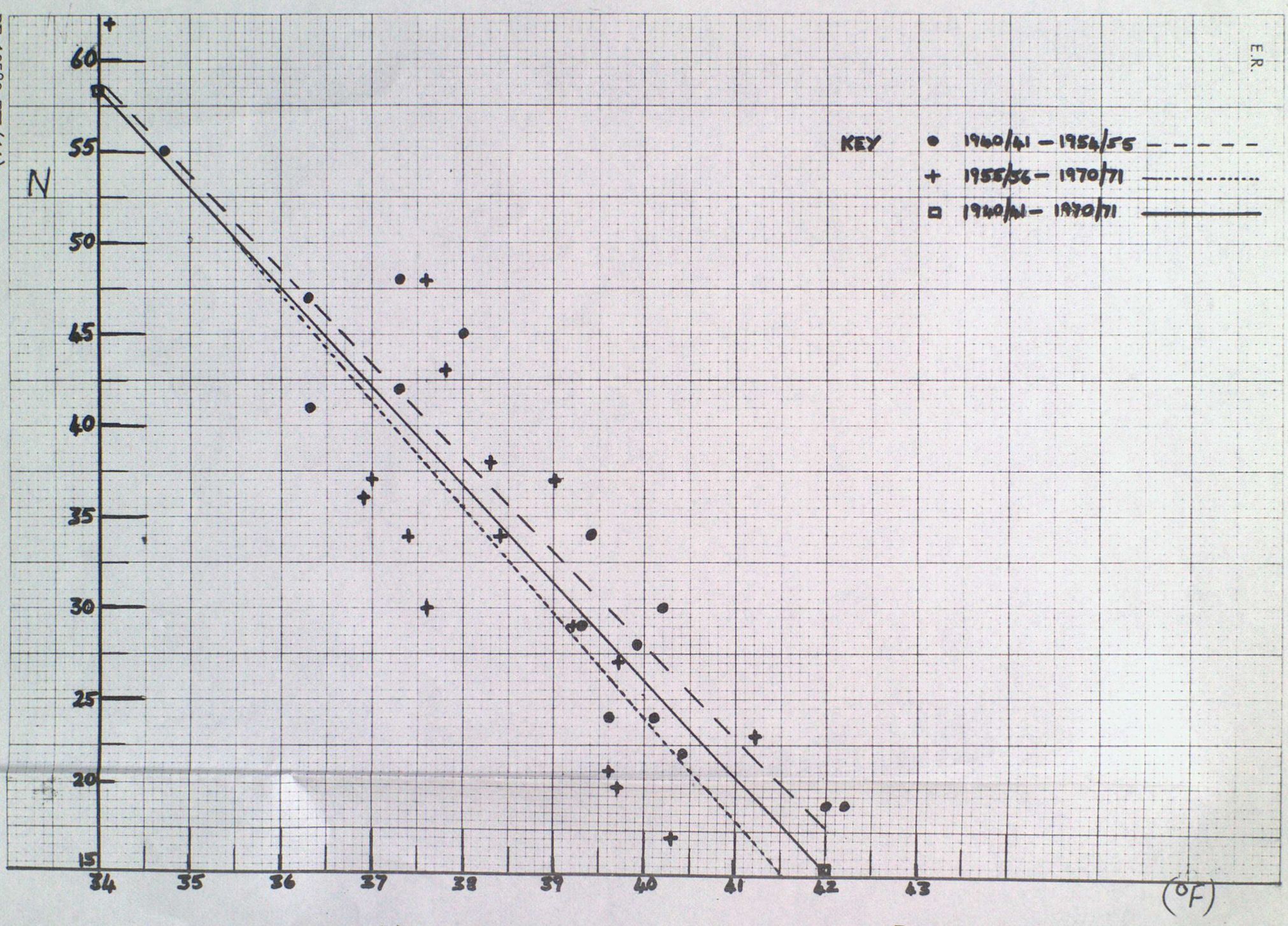

Fig 2 Number (N) of air frosts against mean daily air temperature ($\bar{T}$) for the winter period (December–February incl) and best fitting linear relationships. Edinburgh (Blackford Hill Observatory) 1940/41 to 1970/71.

RE 19589 150 3/73 TEL(13)

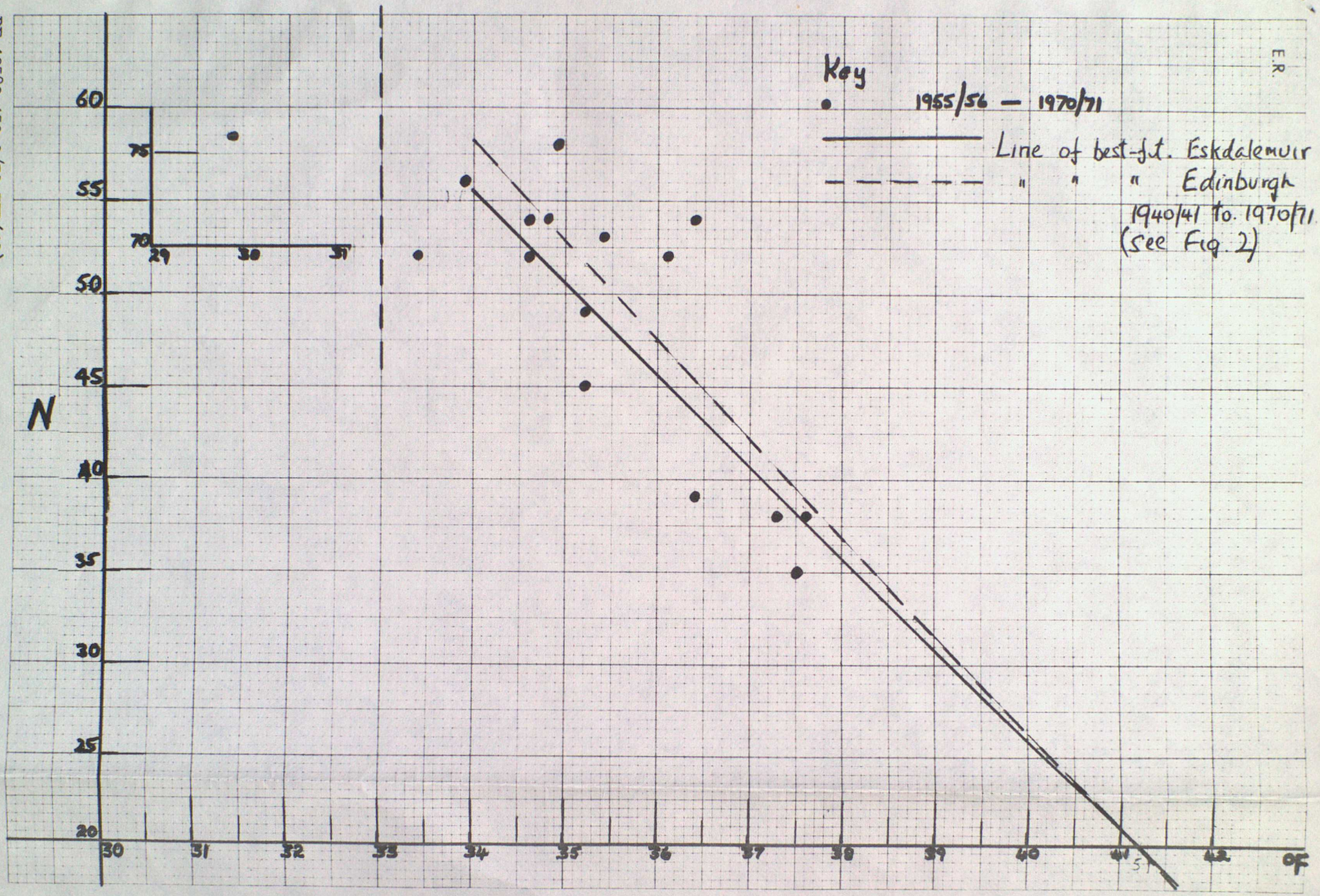

Fig 3 Number (N) of air frosts against mean daily air temperature ($\bar{T}$) for the winter months (December–February incl) and best fitting linear relationship for Eskdalemuir Observatory 1955/56 to 1970/71; and, for comparison, linear regression for Edinburgh.